Productive demand and sectoral capacity utilization

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Introduction

Motivation: demand, slack, and measured productivity

- Central question: what drives business cycles?
- Long-standing debate on technology vs. demand shocks is inseparable from two related puzzles:
  1. **Endogeneity of TFP:** Evans (1992) shows demand-side variables Granger-cause the Solow residual; Basu, Fernald, and Kimball (2006) find purified technology is half as volatile and largely acyclical
  2. **Procyclicality of capacity utilization:** The Fed's capacity utilization rate closely tracks the cycle—but what drives it?
- This paper: goods market frictions provide a unified mechanism linking demand, utilization, and measured TFP

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Proposed explanation via goods market frictions

- Goods market frictions: households expend *shopping effort* d to match with suppliers
  - Increases in shopping effort generate more matches and raise capacity utilization
  - Higher utilization raises measured TFP and output $\Rightarrow$ demand-determined output *without* nominal rigidity
- Estimate multisector model by Bayesian methods on sectoral and capacity utilization data

Key contributions

- **Quantitative:** Search demand shocks account for $\approx 70\%$ of output variance and $\approx 46\%$ of TFP variance—without nominal rigidity
- **Identification:** Capacity utilization data pins down novel shopping-effort parameters ϕ and η precisely; falsifiable test of the mechanism
- **Decomposition:** Sectoral TFP growth $=$ utilization $+$ technology $+$ input-share mismeasurement
- **Comovement:** Sector-specific wage markups are essential to jointly fit sectoral labor and utilization

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- Bai, Rios-Rull, and Storesletten (2025) (BRS): goods market frictions ⇒ productive demand in an estimated model
  - Use *shopping-time data* for calibration and identification; do not use capacity utilization
- **Problems with shopping-time proxy**
  - Matching efficiency shocks are confounded with shopping effort shocks (not separately identified)
  - Shopping time is contaminated with leisure
  - Limited time series: annual data, post-2003 only
  - Low portability: no cross-country equivalent

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- **This paper:** capacity utilization is observable, quarterly (1964–2019), available by sector (durables/non-durables), and does not require constant returns

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Capacity utilization data

- Total capacity utilization from Federal Reserve Board is the ratio of an output index to a capacity index
- Capacity index designed to measure greatest level of output each plant can maintain within realistic work schedule
- Coverage
  - 89 detailed industries (71 manufacturing, 16 mining, 2 utilities)
  - Primarily correspond to industries at the 3 or 4-digit NAICS
  - Estimates are available for non-durables and durables
  - Not available for services
- Responses to the Bureau of the Census's Quarterly Survey of Plant Capacity (QSPC) at establishment level
- Compared to Fernald utilization measure, does not require assuming constant returns to scale/zero profits

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Motivation: Utilization measures and output

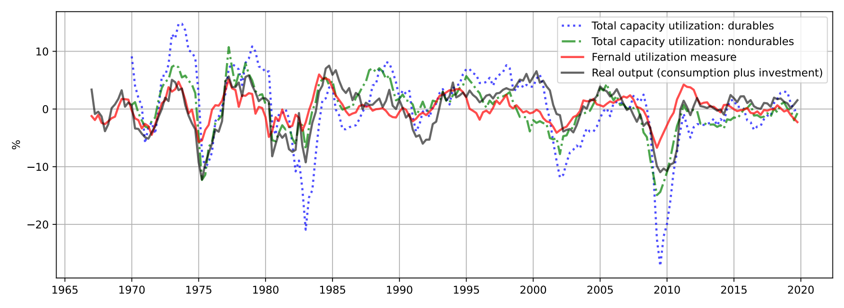

Figure 1: Total capacity utilization (non-durable and durable goods), Fernald utilization, and output. Each underlying series is detrended via the Hamilton regression filter $(p = 4, h = 8)$.

Key data facts

- Capacity utilization in durables and non-durables comove and are strongly procyclical
- Durables utilization is $2\times$ more volatile than non-durables (std 2.27 vs 1.26) $\Rightarrow$ important sectoral discipline on the model

Second moments (growth rates)

Variable	Symbol	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_I)	Cor(x,x$_{-1}$)
Real GDP	Y	0.87	1.00	0.94	0.70	0.47
Real Consumption	C	0.44	0.51	0.54	0.44	0.48
Real Investment	I	2.14	2.46	1.00	0.73	0.41
Labor in Consumption	n_c	0.57	0.66	0.66	0.87	0.67
Labor in Investment	n_i	1.94	2.23	0.73	1.00	0.64
Labor productivity	Y/N	0.64	0.73	0.36	-0.28	0.10
Relative price of investment	p_i	0.51	0.58	-0.28	-0.22	0.44
Utilization in Durables	$util_d$	2.27	2.61	0.69	0.84	0.55
Utilization in Non-durables	$util_{nd}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment. Following Katayama and Kim (2018), we construct labor in consumption as sum of labor in nondurables and services and labor in investment as sum of labor in construction and durables.

Related literature

1. **Purifying the Solow residual:** Basu, Fernald, and Kimball (2006), Fernald (2014)

   ⇒ We use their decomposition as the identification target for demand's role in TFP

2. **Goods market frictions and productive demand:**

   Moen (1997), Michaillat and Saez (2015), Bai, Rios-Rull, and Storesletten (2025), Huo and Ríos-Rull (2018), Qiu and Ríos-Rull (2022), Petrosky-Nadeau and Wasmer (2015), Bethune, Rocheteau, and Rupert (2015), Borys, Doligalski, and Kopiec (2021), Sun (2024)

   ⇒ We extend to a multisector estimated DSGE using capacity utilization data (not shopping time)

3. **Sectoral comovement and factor mobility:**

   Long and Plosser (1983), Christiano and Fitzgerald (1998), Horvath (2000), Katayama and Kim (2018)

   ⇒ Imperfect labor mobility + sector-specific wage markups essential for joint fit

4. **Capacity utilization and business cycles:**

   Corrado and Mattey (1997), Christiano, Eichenbaum, and Trabandt (2016), Qiu and Ríos-Rull (2022)

   ⇒ First to use sectoral (durable/non-durable) utilization data for Bayesian identification

Production model with shocks and dynamics

Production technology

- 2 consumption sectors (nondurable goods mc and services sc) and an investment sector
- Each uses capital k and labor n to produce output
- Stochastic trend to technology X: growth rate $g_t = X_t / X_{t-1}$ is stationary
- Stationary technology component z_j
- Potential output given capital utilization rate h and fixed cost $\nu_j X$.

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\}, \quad (z_{mc} = z_{sc} \equiv z_c)$$

for

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k}, \; \alpha_k + \alpha_n \leq 1$$

- Set $z_i = z_c z_I$, where z_I is independent of z_c

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Investment

- Households shop for investment goods, accumulate capital in each sector, and collect rental income
- Capital law of motion in each sector

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

where $i = i_{mc} + i_{sc} + i_i$

- Endogenous capital depreciation (Christiano, Eichenbaum, and Trabandt (2016))

$$\delta^j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2, \quad j \in \{mc, sc, i\}, \sigma_{ac} \equiv \sigma_{amc} = \sigma_{asc}$$

$\Rightarrow \sigma_{aj} = \delta''_j(1)/\delta'_j(1)$ is the elasticity of marginal utilization cost wrt h at $h = 1$

- Investment adjustment cost (Christiano, Eichenbaum, and Evans (2005))

$$S(x) = \frac{\Psi_K}{2}(x-1)^2$$

Matching technology

- Each market j is subject to Cobb-Douglas matching function

$$M_j(D,T) = A_j D^{\phi} T^{1-\phi}$$

where D is aggregate shopping effort and T is the measure of suppliers

- Normalize $T = 1$ so that D describes market tightness (search effort per supplier)
- Implied matching rates:

$$\Psi_{jd}(D) = M/D = A_j D^{\phi-1}, \quad \Psi_{jT}(D) = M/T = A_j D^{\phi}$$

- Competitive search: households shop in markets indexed by (p_j, D_j, F_j): price, tightness, and quantity

Realized output $<$ Potential Output $\Rightarrow$ capacity utilization

$$\underbrace{y_j}_{\text{value added}} = \overbrace{d_j}^{\text{search intensity}} \times \underbrace{\Psi_{jd}(D)}_{\text{matching prob.}} \times \overbrace{F_j}^{\text{potential output}}, \quad \underbrace{util_j \equiv \frac{y_j}{F_j}}_{\text{endogenous}} = d_j \Psi_{jd}(D_j)$$

Preferences

- Households have GHH preferences over search effort, consumption, and a labor composite

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1 - \sigma}$$

where Γ is a composite parameter with external habit formation:

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1+1/\zeta}$$

- Aggregate consumption C and total search effort $d = d_{mc} + d_{sc} + \theta_i d_i$
- Preference shifters $\theta = \{\theta_b, \theta_d, \theta_i, \theta_n\}$

  $\Rightarrow$ level consumption shock θ_c could be replicated by a proportional decrease in θ_d and θ_n and change in β

Novel demand shocks (shopping effort shocks)

θ_d and θ_i are novel demand shocks given goods market frictions

Consumption aggregator

- Consumption is bundle of non-durable goods y_{mc} and services y_{sc}

$$c = [\omega_{mc}^{1-\rho_c} y_{mc}^{\rho_c} + \omega_{sc}^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c} \tag{1}$$

  such that $\omega_{mc} + \omega_{sc} = 1$

- Price index

$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}} \tag{2}$$

- Normalize $p_c = 1$

Imperfect labor mobility across sectors

- Assume imperfect substitutability between labor used in consumption and investment sectors (Horvath (2000) and Katayama and Kim (2018))

$$n^a = \left[\omega^{-\varepsilon} n_c^{1+\varepsilon} + (1-\omega)^{-\varepsilon} n_i^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$$

- Labor composite n^a measures *perceived* labor hours by worker
- Elasticity of substitution $1/\varepsilon$ measures intersectoral labor mobility
- Induces wage dispersion
- As $\varepsilon \to 0$, $n^a \to n_c + n_i = n$ (perfect mobility benchmark)
- For ε fixed, if $\omega = n_c/n$, then $n^a = n_c + n_i = n$

Differentiated labor and labor unions

- Continuum of monopolistically competitive labor unions in sector j provide services to firms
- Total labor is a CES aggregate of specialized types

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

- Pay workers W^* per unit and rent to firms at rate $W(s)$
- Wage paid by firm is a markup of (variable) wage received by workers

$$W_j = \mu_j W_j^*$$

  with difference $W_j - W_j^*$ rebated to HH as fixed wage
- Let π be sum of profits and wage rebate $\sum_j W_j - W_j^*$ received by households

Role of different ingredients

Ingredient	Role	Reference
Capital intensity	Control/amplification of technology shocks	Christiano, Eichenbaum, and Evans (2005)
Imp. factor mobility	Sectoral comovement	Horvath (2000), Katayama and Kim (2018)
Habit formation	Smooth consumption response	Christiano, Eichenbaum, and Evans (2005)
Inv. Adjustment costs	Hump-shaped investment responses	Christiano, Eichenbaum, and Evans (2005)
Fixed costs	Control (procyclical productivity)	Christiano, Eichenbaum, and Trabandt (2016)
Labor unions	Sectoral wage markups and shocks	Schmitt-Grohé and Uribe (2012)

Table 2: Compact overview of ingredients and their roles

Timing

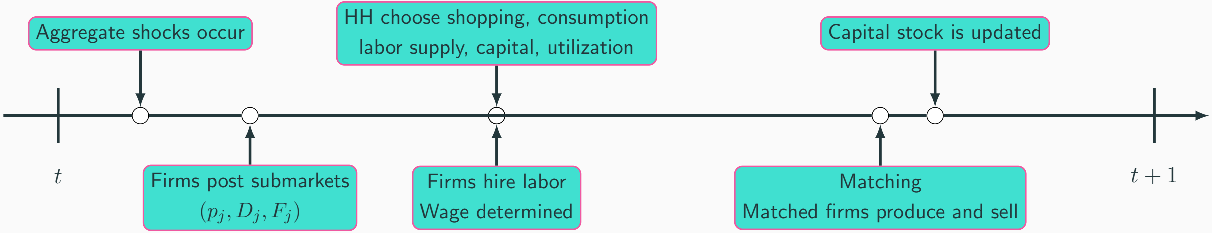

Figure 2: Timing

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates given markets $(p_j, D_j, F_j), j \in \{mc, sc, i\}$ and the aggregate state of the economy Λ

$$\widehat{V}(\Lambda, \{k_j\}, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k_j', h_j'} \{u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', \{k_j'\})|\Lambda\}\}$$

s.t.

$$y_j = d_j \Psi_{jd}(D_j) F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_{j\in\{mc,sc,i\}} y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c^* + n_i W_i^*$$

$$k_j' = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

subject to endogenous depreciation δ_j, investment adjustment cost S_j, and consumption and labor aggregators (1) and (2)

- The value function is determined by the best market:

$$V(\Lambda, \{k_j\}) = \max_{\{p,D,F\}\in\Phi} \widehat{V}(\Lambda, \{k_j\}, p, D, F)$$

Details of household problem

Competitive search: the shopping wedge ϕ

- FOC with respect to consumption output and shopping effort yield

$$u_d = -A_j D_j^{\phi-1} F_j (u_j - \lambda p_j)$$

- Note that HH cares about difference between marginal utility and price
- Divide by u_j to express this in terms of the shopping wedge

$$-\frac{u_d}{u_j} = A_j D_j^{\phi-1} F_j \frac{u_j - \lambda p_j}{u_j}$$

Lemma 1 (Elasticity as shopping wedge)

The matching elasticity ϕ equals relative wedge between marginal utility and price in equilibrium:

$$\phi = \frac{u_j - \lambda p_j}{u_j}$$

Optimal shopping effort and demand

Optimal shopping effort: consumption sectors

$$\overbrace{-\frac{u_d}{u_j}}^{\text{MRS}} = \overbrace{\phi\Psi_{jd}(D_j)}^{\Psi'_{jT}(D)} F_j \quad j \in \{mc, sc\}$$

- MRS = MRT: marginal shopping cost equals marginal gain from raising firm match rate × output
- Under GHH preferences: $\theta_d D^{1/\eta} = \phi A_j D_j^{\phi-1} F_j$

Investment shopping (converted to consumption units via p_i/p_{mc})

$$-\frac{u_d}{u_{mc}}\theta_i = \frac{p_i}{p_{mc}}\phi A_i D_i^{\phi-1} F_i$$

- Demand curves for non-durables and services: $y_j = p_j^{-\xi}\omega_j C$, $\xi = 1/(1-\rho_c)$

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Firms' problem

- A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets
- Firm chooses labor, capital inputs and submarket (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, F_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$z_j f(h_j k_j, n_j) - \nu_j X \geq F_j$$

$$\widehat{V}(\Lambda, \{k_j\}, p_j, D_j, F_j) \geq V(\Lambda, \{k_j\})$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Details of firm problem

Firm factor demands and relative price of investment

Firm factor demands

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\}$$

1. Input demand depends positively on shopping effort (multiplier effect)
2. Matching function elasticity ϕ appears as separate factor (from competitive search)
   $\Rightarrow$ Additional output relaxes participation constraint of households and effectively reduces input cost

Relative shopping effort

A simple static model

Static intuition: how demand raises measured TFP

- Strip to essentials: no investment, homogeneous labor, $F = zn^{\alpha_n}$

Shopping (HH FOC)	$\theta_d D^{1/\eta} = \phi C/D$
Output (matching)	$C = AD^{\phi} F$
Labor demand	$(1-\phi)W = \alpha_n C/n$
Labor supply	$\theta_n n^{1/\zeta} = (1-\phi)W$

- Labor share $\tau \equiv Wn/C = \alpha_n/(1-\phi) > \alpha_n$ (shopping wedge raises apparent labor share)
- Solow residual $SR \equiv C/n^{\tau} = AD^{\phi} zn^{-\alpha_n \phi/(1-\phi)}$
  $\Rightarrow$ demand shock $\uparrow D \Rightarrow \uparrow SR$ even with z fixed

Equilibrium in static setting

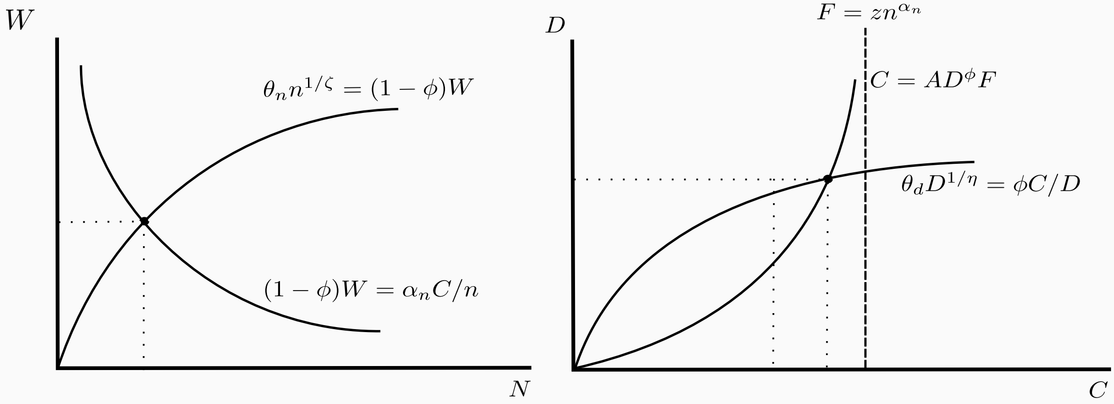

Figure 3: Equilibrium of static model

$$D_1 = [C/(AF)]^{1/\phi} \quad \text{(Consumption production)}, \quad D_2 = [\phi C/(\theta_d)]^{\eta/(\eta+1)} \quad \text{(Shopping)}$$

Demand shock: reduction in θ_d

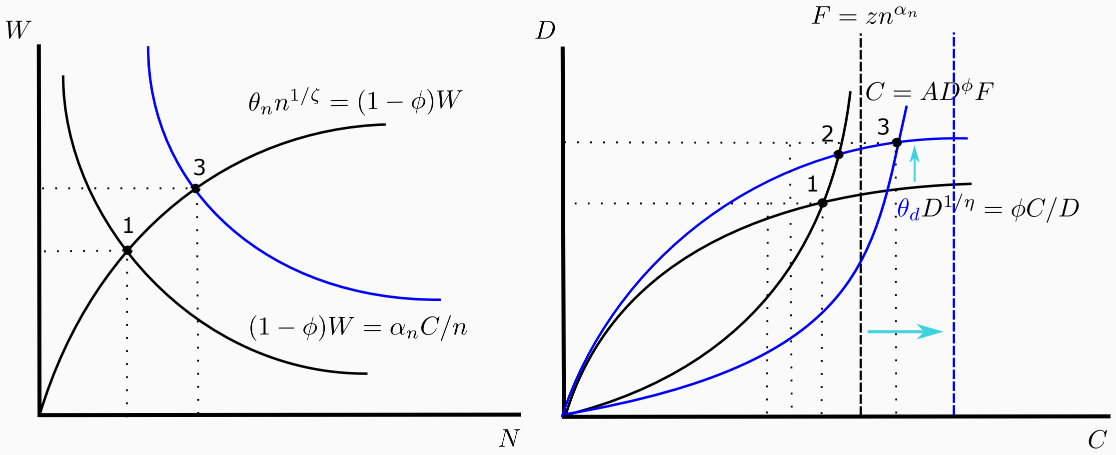

Figure 4: Reduction of shopping disutility in static model

Growth rates of Solow residual and capacity utilization

Sectoral Solow residual, capacity and utilization

- Define sectoral Solow residual, capacity, and capacity utilization following Qiu and Ríos-Rull (2022)

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}}, \quad cap_{jt} = z_{jt} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n} X_t^{1-\alpha_k} - \nu_{jt} X_t, \quad util_{jt} \equiv \frac{Y_{jt}}{cap_{jt}}$$

  given steady-state labor income share $\tau = \alpha_n (1 + \nu_{ss}^R)/(1-\phi)$ and $\nu_{ss}^R = \nu_j X/(z_j f - \nu_j X)$

- Let $dx_t \equiv \log(x_t/x_{t-1})$ denote growth rate

Growth rate decomposition

$$dSR_{jt} = \overbrace{\phi dD_{jt}}^{\text{Shopping}} + \underbrace{\alpha_k dh_{jt}}_{\text{Capital intensity}} + \overbrace{dz_{jt} + (1-\alpha_k) dX_t}^{\text{Technology}} + \underbrace{(\alpha_k - 1 + \tau) dk_{jt} + (\alpha_n - \tau) dn_{jt}}_{\text{Input share mismeasurement}} + \overbrace{d(1+\nu_{jt}^R)}^{\text{Fixed costs}}$$

$$dutil_{jt} = \phi dD_{jt} + (1 + \nu_{ss}^R) \alpha_k dh_{jt}$$

$$dSR_{jt}|_{\nu_j = 0} = \overbrace{dutil_{jt}}^{\text{Utilization}} + \underbrace{dz_{jt} + (1-\alpha_k) dX_t}_{\text{Technology}} + \overbrace{(\alpha_k - 1 + \tau) dk_{jt} + (\alpha_n - \tau) dn_{jt}}^{\text{Input share mismeasurement}}$$

Aggregate measures

- Output

$$Y = C + p_i^{ss} I$$

- Using base-year prices makes results independent of numeraire choice Explanation
- Solow residual and capacity utilization

$$SR = \prod_j SR_j^{\frac{Y_j}{Y}}, \quad util = \prod_j util_j^{\frac{Y_j}{Y}} \tag{3}$$

- Applying logs and first differencing to (3) immediately implies

$$dSR = \sum_j \frac{Y_j}{Y} dSR_j, \quad dutil = \sum_j \frac{Y_j}{Y} dutil_j$$

Equilibrium Labor share derivation

Quantitative analysis of general model

Stochastic processes

- The growth rate of the stochastic trend $g_t = X_t/X_{t-1}$ follows an AR(1) process in logs as BRS

$$\log g_t = (1-\rho_g)\log \overline{g} + \rho_g \log g_{t-1} + e_{g,t}$$

where $e_{g,t} \sim N(0, \sigma_g)$

- Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}$$

where $e_{v,t} \sim N(0, \sigma_v)$

- Stationarize trending variable by dividing by X_t (X_{t-1} in case of predetermined capital stock K_{jt})

Observation equations

Balanced growth path

Bayesian estimation and choice of observable variables

- Parameter space Θ and data $\mathcal{Y}$
- Bayesian estimate model subject to steady-state targeting
- Sample from posterior distribution combining likelihood and prior

$$P(\Theta|\mathcal{Y}) = \frac{L(\mathcal{Y}|\Theta)P(\Theta)}{P(\mathcal{Y})}, \quad P(\mathcal{Y}) = \int L(\mathcal{Y}|\Theta)P(\Theta)d\theta$$

- Use random-walk MH to sample posterior

Seven observables in growth rates: $1964Q1 - 2019Q4$

$$\mathcal{Y}_t = \begin{bmatrix} dC_t & dI_t & dn_{ct} & dn_{it} & dutil_{ND,t} & dutil_{D,t} & dp_{it} \end{bmatrix}'$$

- Use sectoral data on output and labor following Katayama and Kim (2018)
- Construct output from sum of private consumption and private investment (as BRS)
- Note that sectoral dataset implicitly targets labor productivity in each sector

Proof of concept using BRS model Estimation procedure

Calibration: exogenous and estimated parameters

Targets	Target value	Parameter	Calibrated value/posterior mean
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average annual growth rate	1.8%	$\bar{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.58
Frisch elasticity	–	ζ	1.24
Elasticity of matching function	–	ϕ	0.92
Elasticity of shopping effort cost	–	η	0.22
Fixed cost share of capacity	–	ν^R	0.094
Habit persistence	–	ha	0.74

Table 3: Calibration: exogenous and estimated parameters

Posterior estimates: structural parameters

		Prior			Posterior			
Par	Interpretation	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	Risk aversion	Beta	1.50	0.250	1.59	0.240	1.23	1.98
ha	Habit formation	Beta	0.500	0.200	0.739	0.0322	0.692	0.792
ζ	Frisch elast. of labor supply	Gamma	0.720	0.250	1.26	0.215	0.947	1.60
ϕ	Elast. of matching	Beta	0.320	0.200	0.917	0.0531	0.834	0.991
η	Elast. shopping disutility	Gamma	0.200	0.150	0.221	0.0388	0.159	0.277
ξ	Cons. elast. of subs.	Gamma	0.850	0.100	0.880	0.0826	0.753	1.02
ν_R	Fixed cost share	Beta	0.200	0.100	0.090	0.0400	0.028	0.152
σ_{ac}	Depreciation elast:cons	Inv. Gamma	1.00	1.00	1.74	0.328	1.20	2.23
σ_{ai}	Depreciation elast:inv	Inv. Gamma	1.00	1.00	0.440	0.0893	0.298	0.580
Ψ_K	Inv. adj. cost	Gamma	4.00	1.00	12.6	1.60	10.5	15.5
ε	Inv. elast. of labor mobility	Gamma	1.00	0.500	1.46	0.218	1.10	1.82

Table 4: Posterior estimates: structural parameters

Posterior and prior density: ϕ and η

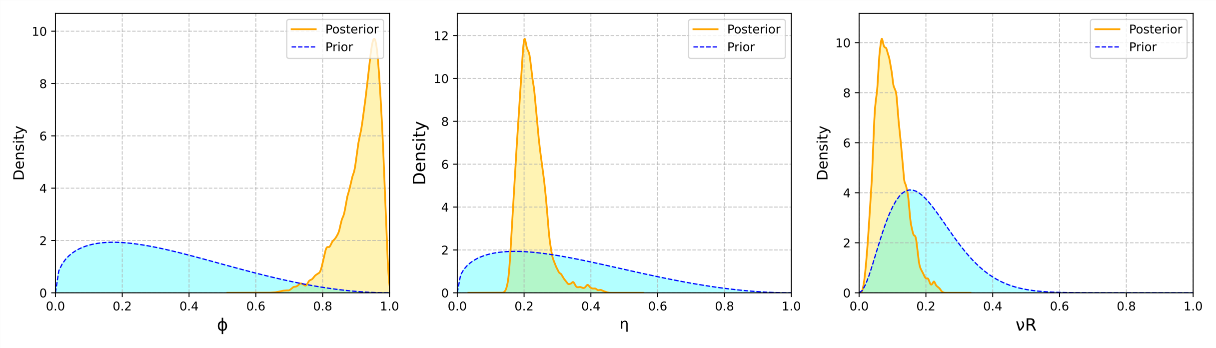

Figure 5: Posterior and prior distributions for matching function elasticity ϕ, the shopping disutility parameter η, and the fixed cost share ν^R.

Estimation on artificial data (parameter values set to posterior mean)

		Posterior distribution		
Parameter	True value	Median	5%	95%
σ	1.58	1.70	1.24	1.85
ha	0.736	0.714	0.689	0.746
ζ	1.24	1.29	1.09	1.50
ϕ	0.913	0.911	0.872	0.955
η	0.224	0.241	0.203	0.276
ξ	0.882	0.809	0.717	0.892
ν^R	0.0943	0.118	0.0694	0.21
σ_{ac}	1.76	2.04	1.69	2.47
σ_{ai}	0.441	0.289	0.216	0.374
Ψ_K	12.5	7.78	7.26	8.30
ε	1.46	1.53	1.38	1.69

Table 5: Parameters well identified in exercise using artificial data generated from model evaluated at posterior mean (Schmitt-Grohé and Uribe (2012))

Unconditional forecast error variance decomposition: grouped shocks

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	28.6	0.01	70.5	0.92	0.02
SR	44.3	5.23	46.0	0.57	3.9
I	31.2	0.01	64.1	4.69	0.01
p_i	65	0.00	34.8	0.18	0.05
n_c	7.78	27.2	58.4	4.42	2.19
n_i	18.2	2.27	52.9	1.76	24.8
$util$	39.3	0.01	60.1	0.64	0.01
D	0.17	0	99.8	0.01	0
h	17.7	0.01	82.1	0.18	0

Table 6: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Breakdown of demand and technology shocks

Model comparison

	Data	Baseline	Perfect labor mobility	Common wage markup	Remove: Fixed cost	Remove: VCU	Remove: SDS	Remove: SDS and utilization data
LML	–	4570.7	4548.9	3136.5	4573.4	4,568.1	2564.9	–
Δ LML	–	0	-21.9	-1434.2	2.71	-2.6	-2006	–
Posterior mean ϕ	–	0.91	0.43	0.95	0.96	0.27	0.71	0.52
FEVD(SR, SDS)	–	46.0	55.0	6.17	45.71	48.07	–	–
Var$(util)$/Var(SR)	–	1.95	1.21	0.40	2.36	0.56	2.21	0.19
std(Y)	0.87	1.38	1.70	5.11	1.36	1.99	207.71	0.64
std$(util_{ND})$	1.26	1.21	1.30	3.88	1.22	1.21	161.65	0.35
std$(util_D)$	2.27	3.65	2.60	9.72	3.90	2.37	266.65	1.14
std(n_c)	0.57	0.66	0.68	2.27	0.68	0.69	71.31	0.56
std(n_i)	1.94	2.35	3.33	8.99	2.36	1.80	344.8	1.87
Cor(C, I)	0.54	0.53	0.67	0.05	0.50	0.53	0.999	0.24
Cor$(util_{ND}, util_D)$	0.75	0.27	0.61	-0.31	0.26	0.60	0.999	-0.60
Cor(n_c, n_i)	0.87	0.67	0.24	-0.88	0.69	0.35	0.986	0.83
Cor$(util_{ND}, util_{ND,-1})$	0.51	0.21	0.31	0.53	0.18	-0.02	0.999	0.27
Cor$(util_D, util_{D,-1})$	0.55	0.48	0.52	0.51	0.48	0.03	0.999	0.26

Table 7: Comparison of model specification

Takeaway from model comparison

1. **Search demand shocks are essential** (ΔLML $= -2006$)
   $\Rightarrow$ Without SDS, capacity utilization data pins down sectoral shopping effort—model cannot jointly fit sectoral labor, output, and the relative price of investment
2. **Sectoral wage-markup shocks are essential** (ΔLML $= -1434$)
   $\Rightarrow$ Without them, shopping effort is mechanically tied to the labor ratio, losing flexibility to match utilization comovement
3. **Baseline fits sectoral data well** despite modest overstatement of output volatility and understatement of utilization comovement
   $\Rightarrow$ Fixed costs and variable capital utilization are secondary: $|\Delta\text{LML}| \leq 3$

IRF: negative e_D shock (lower shopping disutility $\Rightarrow$ demand boom)

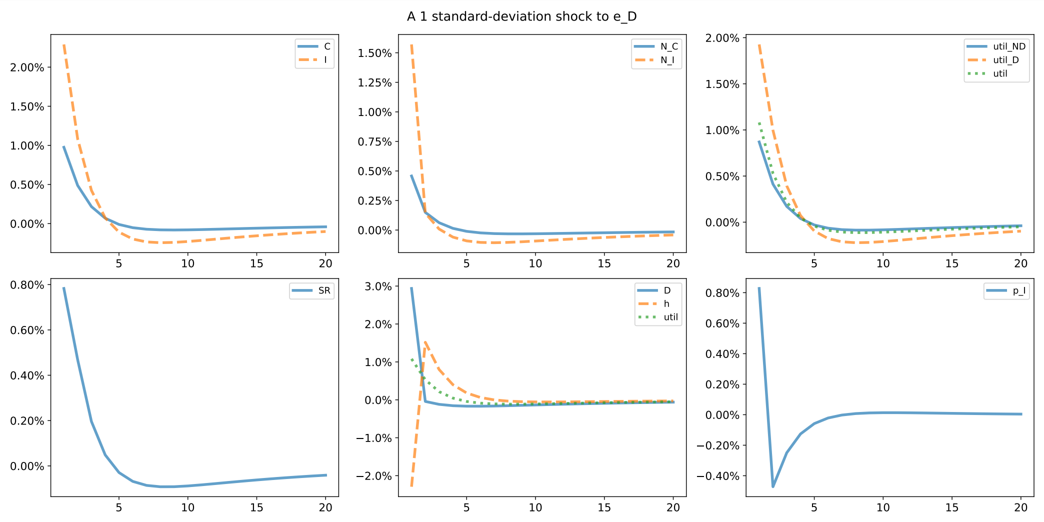

Figure 6: The vertical axis measures response in growth rates.

IRF: positive e_z shock (neutral technology) — contrast with e_D

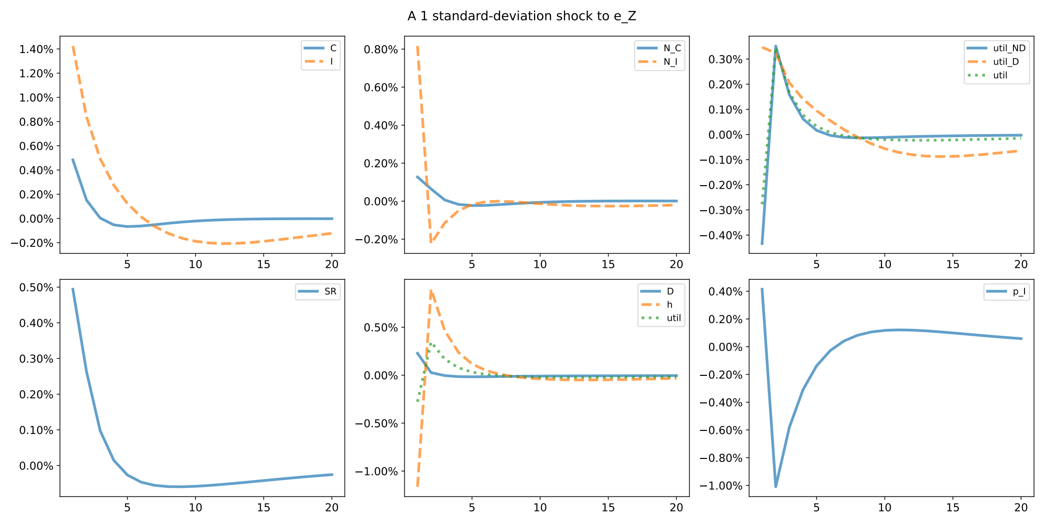

Figure 7: The vertical axis measures response in growth rates.

IRF: positive e_b shock (discount factor) — C vs. I tradeoff

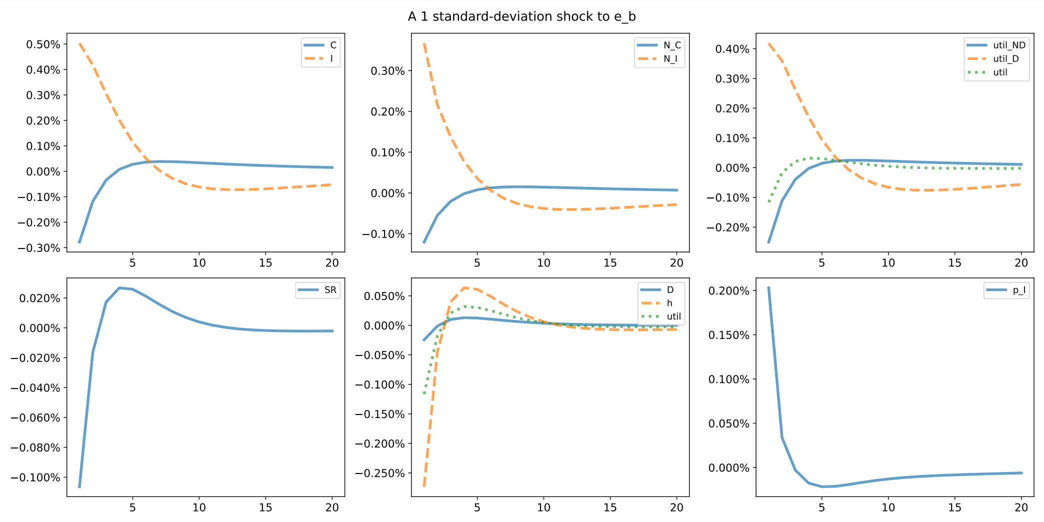

Figure 8: The vertical axis measures response in growth rates.

Conclusion

1. **Main finding:** Search demand shocks account for $\approx 70\%$ of output variance and $\approx 46\%$ of TFP variance in a multisector model estimated by Bayesian methods—no nominal rigidity required
2. **Identification:** Sectoral capacity utilization data (durables and non-durables, 1964–2019) pins down shopping-effort parameters $\phi \approx 0.92$ and $\eta \approx 0.22$ precisely; artificial-data exercise confirms recovery
3. **Decomposition:** Sectoral TFP growth $=$ utilization (shopping) $+$ technology $+$ input-share mismeasurement—provides a model-consistent analog to Fernald (2014)
4. **Model architecture:** Sector-specific wage markups are essential (ΔLML $= -1434$); fixed costs and variable capital utilization are secondary
5. **Open directions:** Extend to labor market frictions, cross-country data, and linking shopping activity to confidence or liquidity shocks

Extensions

- Relax competitive search–implies efficiency and has strong implication for labor share of income
  $\Rightarrow$ alternatively, entry margin gives rise to inefficiency even under competitive search
- Link shopping activity to confidence shocks (Angeletos, Collard, and Dellas (2018)) or liquidity shocks
- Estimate version with labor market frictions to endogenously explain imperfect intersectoral mobility
- Adapt approach to other countries: need to disaggregate utilization into durables and non-durables
- Consider firms shopping for investment goods using expenditure in terms of labor

Appendix

Labor share of income

- Labor share of income is key component to constructing Solow residual
- Define fixed cost share $\nu_j^R = \nu_j X/(z_j f - \nu_j X)$, so that $Y_j = A_j D_j^\phi z_j f/(1+\nu_j^R)$
- Write sectoral labor share of income as

$$\frac{W_j n_j}{p_j Y_j} = \frac{\alpha_n(1+\nu_j^R)}{1-\phi}$$

- Provided $\nu_j^R = \nu^R$ for all j, overall steady-state labor share of income is

$$\tau \equiv \frac{Wn}{Y} = \frac{\alpha_n(1+\nu^R)}{1-\phi}$$

Relative shopping effort under zero fixed costs

- Under zero fixed costs, combining firm investment relative price with HH optimal shopping yields

$$\frac{D_i}{D_j} = \frac{1}{\theta_i}\frac{n_i W_i}{n_j W_j}$$

- Relative shopping effort closely tied to relative labor hours
  $\Rightarrow$ reduces to $D_i/D_c = n_i/n_c$ in BRS
- But shopping effort ratio also needs to match relative capacity utilization $\Rightarrow$ role for wage dispersion

Data series

ID	Description	Source
PCND	Personal consumption: non-durable	BEA
PCESV	Personal consumption: services	BEA
HOANBS	Nonfarm business hours worked	BLS
CPIAUCSL	Consumer price index	BLS
GDPC1	Real GDP	BEA
GDPIC1	Real gross private domestic investment	BEA
COMPRNFB	Wages (real compensation per hour)	BLS
CNP160V	Civilian non-institutional population	BLS
GDPDEF	GDP Deflator	BEA
SR	Solow residual	Fernald (2014), FRB of San Francisco
Util	Total capacity utilization	Federal Reserve Board of Governors
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Back to second moments

Construction of variables

Symbol	Description	Construction
C	Nominal consumption	$PCEND + PCESV$
I	Nominal gross private domestic investment	GPDI
Deflator	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
c	Real per capita consumption	$\frac{C}{Pop*P_c}$
i	Investment	$\frac{I}{Pop*P_i}$
y	Real per capita output	$c + i$
N_c	Labor in consumption sector	Labor in nondurables and services, BLS
N_i	Labor in investment sector	Labor in construction and durables, BLS
N	Aggregate labor	$N_c + N_i$
P_i	Price index: investment goods	$A006RD3Q086SBEA$
P_c	Price index: consumption goods	$DPCERD3Q086SBEA$
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: non-durables	Federal Reserve Board
$util_D$	Total capacity utilization: durables	Federal Reserve Board
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR$_{util}$	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

More details on construction of sectoral data

- Closely follows Katayama and Kim (2018)
- Construct consumption and investment as follows

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)} \right)$$

$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)} \right)$$

- Use HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates
- P_c: combine price indices of nondurable goods (DNDGRG3Q086SBEA) and services (DSERRG3Q086SBEA)
- P_i: use quality-adjusted investment deflator (INVDEV)

More details on construction of sectoral data

- BLS Current Employment Statistics (https://www.bls.gov/ces/data)
- BLS Table B6 contains the number of production and non-supervisory employees by industry
- BLS Table B7 contains average weekly hours of each sector
- We compute total hours for non-durables, services, construction, and durables by multiplying the relevant components of each table
- Construct labor in consumption as sum of non-durables and services
- Construct labor in investment as sum of construction and durables

Related international measures of capacity utilization

Country	Sectors Covered	Survey/ Calc.?	How Question is Framed / Method	Source
Canada	Manufacturing, mining, utilities, construction	Survey	"At what percentage of your production capacity are you currently operating?"	Statistics Canada
Euro Area	Manufacturing	Survey	"At what capacity is your company currently operating (as a percentage of full capacity)?"	European Commission
UK	Manufacturing, services	Survey	"What is the current rate of capacity utilization in your business?" (firms give a percentage)	Office for National Statistics / CBI
Japan	Manufacturing, mining	Calculated	Based on indices of industrial production and capacity, using statistical/engineering estimates	Ministry of Economy, Trade and Industry (METI)
South Korea	Manufacturing	Survey	Firms are surveyed: "At what % of capacity are you currently operating?"	Statistics Korea (KOSTAT)
Russia	Manufacturing	Survey	Firms report their current use of production capacity as % of "normal/full" capacity	Rosstat
China	Manufacturing	Survey	Firms asked: "What is the current utilization rate of your production capacity?"	National Bureau of Statistics of China (NBSC)

Back to Questions and key contribution

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates taking markets $(p_j, D_j, y_j), j \in \{c, i\}$ and the aggregate state of the economy $\Lambda = (\theta, Z, K)$ as given.

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_c, k'_i)|\Lambda\} \quad \text{s.t.}$$

$$y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j \in \{mc, sc, i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

and the consumption and labor aggregators

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p, D, y\} \in \Omega} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, y)$$

First order conditions

- Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_c, \mu_i$ be the respective Lagrangian multipliers on the constraints
- FOC

$$[y_{mc}]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc}$$

$$[y_{sc}]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc}$$

$$[i_c]: \quad -\gamma_i - \lambda p_i + \mu_c \left(1 - S'(x_c)x_c - S(x_c)\right) + \beta\theta_b \mathbb{E}\mu_c' S'(x')(x')^2 = 0$$

$$[i_i]: \quad -\gamma_i - \lambda p_i + \mu_i \left(1 - S'(x_i)x_i - S(x_i)\right) + \beta\theta_b \mathbb{E}\mu' S'(x_i')(x_i')^2 = 0$$

$$[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\}$$

$$[d_i]: \quad u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i$$

$$[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^*$$

$$[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^*$$

$$[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\}$$

$$[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\}$$

Envelope conditions

- Consumption

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{4}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi - 1) d_j A_j D_j^{\phi-2} F_j (u_j - \lambda p_j) \quad j \in \{mc, sc\} \tag{5}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} (u_j - \lambda p_j) \quad j \in \{mc, sc\}$$

- Investment

$$\frac{\partial V^i}{\partial p_i} = -\lambda i = -\lambda (d_i A_i D_i^{\phi-1} F_i) \tag{6}$$

$$\frac{\partial V^i}{\partial D_i} = -(\phi - 1) d_i A_i D_i^{\phi-2} F_i \gamma_i \tag{7}$$

$$\frac{\partial V^i}{\partial F_i} = d_i A_i D_i^{\phi-1} \gamma_i$$

Price-tightness tradeoff

- Take ratio of (4) and (5):

$$\frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)} \tag{8}$$

- Take ratio of (6) and (7)

$$\frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i} \tag{9}$$

Back to household problem

Firms' problem

- A representative firm in sector $j \in \{mc, s, i\}$ rents capital and hires labor in spot markets
- Continuum of monopolistically competitive labor unions in sector j sell differentiated services
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, y_j} p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$\widehat{V}(K, p_j, D_j, F_j) \geq V(K)$$

$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Conditional labor demand and wage index

- Consider labor cost minimization problem

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.}$$
$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj\right)^{\mu_j} \geq \overline{n}$$

- Take FOC and recognize W_j as Lagrangian multiplier on constraint

$$n_j(s) = \left(\frac{W_j(s)}{W_j}\right)^{-\frac{\mu_j}{\mu_j - 1}} n_j \tag{10}$$

- Wage index for composite labor input in sector j

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds\right]^{\mu_j - 1}$$

Optimal wage choice of labor union and aggregation

- Problem of labor union

$$\max_{W_j(s)} (W_j(s) - W_j^*) n_j(s) \quad \text{s.t.} \quad (10) \Leftrightarrow$$

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

- Labor union in each sector choose

$$W_j(s) = \mu_j W_j^*$$

- Labor unions pay same wage and firms choose identical quantities of labor within j

$$W_j(s) = W_j, n_j(s) = n_j$$

- Labor unions rebate earnings to HH in lump-sum fashion (regard as fixed component to wage)

Firm first order conditions

- Let ι_j and ∇_j be the multipliers on participation constraint and production technology

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$[n_j] \quad W_j = \nabla_j z_j f_n$$

$$[k] \quad h_j R_j = \nabla_j z_j f_k$$

$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \tag{11}$$

$$[D_j] \quad \phi A_j D_j^{\phi-1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0 \tag{12}$$

Firm problem: finding λ and γ_j

- Take ratio of first order conditions for (11) and (12)

$$\frac{D_j}{\phi p_j} = \frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}}$$

- Plug in (8)

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)}$$

- Simplify

$$\lambda \phi p_j = (1 - \phi)(u_j - \lambda p_j) \Rightarrow$$

$$\lambda = u_j(1 - \phi)/p_j$$

so that

$$\gamma_j = \phi u_j$$

Firm problem: finding γ_i

- Take ratio of first order conditions for (11) and (12) for $j = i$:

$$\frac{D_i}{\phi p_i} = \frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}}$$

- Plug in (9)

$$\frac{D_i}{\phi p_i} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i}$$

- Simplify

$$\begin{aligned}\gamma_i &= \frac{\phi}{1-\phi}\lambda p_i \\ &= \phi \frac{u_j}{p_j} p_i\end{aligned}$$

Simplifying shopping conditions

- Plug in values of γ_j to find

$$-u_d = \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{m_c, s_c\}$$
$$-u_d \theta_i = \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i]$$

- Plug in $\lambda = u_{mc}(1-\phi)/p_{mc}$ to simplify labor-leisure tradeoff

$$u_n \frac{\partial n^a}{\partial n_j} = -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}$$

Demand for non-durables and services

- From the expression for λ we have

$$\frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \Rightarrow \phi = (u_j - \lambda p_j)/u_j$$

- Combine with consumption aggregation and price index to find demand curves

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{m_c, s_c\}$$

where $\xi = 1/(1-\rho_c)$ is the elasticity of substitution.

Tobin's Q

- Solve for value of investment: $j \in \{c, i\}$

$$\lambda p_i + \gamma_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

$$\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

$$\frac{\lambda p_i}{1-\phi} = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)$$

- Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption
- We can rearrange as

$$\frac{p_i}{1-\phi} = Q_j[1 - S'(x_j)x_j - S(x_j)] + \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda}Q_j' S'(x_j')(x_j')^2$$

Tobin's Q

- Rewrite optimal choice of utilization: $j \in \{mc, sc, i\}$

$$\delta_h(h_j)Q_j = R_j$$

- Euler equation

$$Q_j = \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j'h_j'\right] \quad j \in \{mc, sc, i\}$$

Solving for firm multipliers

$$\iota_j = \frac{A_j q_j^{\phi} F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda}$$

$$\begin{aligned}
\nabla_j &= p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j} \\
&= p_j A_j D_j^{\phi} + \frac{A_j D_j^{\phi} \gamma_j}{\lambda} \\
&= p_j A_j D_j^{\phi} + A_j D_j^{\phi} \frac{\phi}{1-\phi} p_j \\
&= A_j D_j^{\phi} \left(p_j + \frac{\phi}{1-\phi} p_j \right) \\
&= \frac{p_j A_j D_j^{\phi}}{1-\phi}
\end{aligned}$$

Back to firm problem

Simplified optimality conditions for firm

$$
\begin{aligned}
(1-\phi)\frac{W_c}{p_j} &= A_j (D_j)^{\phi} z_c f_{N_j} \quad j \in \{m_c, s_c\} \\
\frac{W_c}{R_j} &= \frac{f_{N_c}}{f_{K_c}} \\
(1-\phi)\frac{W_i}{p_i} &= A_i (D_i)^{\phi} z_i f_{N_i} \\
\frac{W_i}{R_i} &= \frac{f_{N_i}}{f_{K_i}}
\end{aligned}
$$

Firm factor demands

$$(1-\phi)\frac{W_c}{p_j} = \alpha_n \frac{Y_j + A_j D_j^{\phi} \nu_j}{N_j} \quad j \in \{mc, sc, i\}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j + A_j D_j^{\phi} \nu_j}{h_j K_j} \quad j \in \{mc, sc, i\}$$

Summary of equilibrium conditions

$$
\begin{aligned}
\theta_n (n^a)^{1/\nu} \left(\frac{n_c}{n^a}\right)^{\theta} \omega^{-\theta} &= (1-\phi)\frac{W_c}{\mu_c \zeta} \\
\theta_n (n^a)^{1/\nu} \left(\frac{n_i}{n^a}\right)^{\theta} (1-\omega)^{-\theta} &= (1-\phi)\frac{W_i}{\mu_i \zeta} \\
n^a &= \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}} \\
\theta_d D^{1/\eta} &= \phi p_j \frac{Y_j}{D_j} \quad j \in \{mc, sc\} \\
\theta_i \theta_d D^{1/\eta} &= \phi p_i \frac{I}{D_i} \\
\frac{p_i}{1-\phi} &= Q_j [1 - S_j'(x_j) x_j - S(x_j)] + \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} Q_j' S'(x_j')(x_j')^2 \\
Q_j &= \beta \theta_b \mathbb{E} \frac{\lambda'}{\lambda} \left[(1-\delta_j(h_j')) Q_j' + R_j' h_j'\right] \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Summary of equilibrium conditions

$$
\begin{aligned}
C &= [\omega_c^{1-\rho_c} Y_{mc}^{\rho_c} + (1-\omega_c)^{1-\rho_c} Y_{sc}^{\rho_c}]^{1/\rho_c} \\
Y_j &= p_j^{-1/(1-\rho_c)} \omega_j C \quad j \in \{mc, sc\} \\
C &= p_{mc} Y_{mc} + p_{sc} Y_{sc} \\
\lambda &= \Gamma^{-\sigma}(1-\phi)
\end{aligned}
$$

Summary of equilibrium conditions

$$
\begin{aligned}
\delta_h(h_j)Q_j &= R_j, \quad j \in \{mc, sc, i\} \\
Y_j &= A_j(D_j)^{\phi}(z_j(h_jK_j)^{\alpha_k}(n_j)^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\} \\
I &= I_c + I_i \\
K_j' &= (1 - \delta_j(h_j))k_j + [1 - S(x_j)]I_j \quad j \in \{mc, sc, i\} \\
(1-\phi)\frac{W_j}{p_j} &= \alpha_n \frac{Y_j + A_j D_j^{\phi}\nu_j}{N_j} \quad j \in \{mc, sc, i\} \\
\frac{W_j}{R_j} &= \frac{\alpha_n}{\alpha_k}\frac{h_jK_j}{n_j} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Explanation of numeraire dependence

- Quantity movements may depend on the numeraire in a multisector model
- Consider positive shock to Z^C: relative price of consumption goods falls
- In terms of the investment good, consumption may fall even though actual units purchased rises
- However, if the consumption good were the numeraire, the investment good instead rises in price, so output rises by more
- Reasoning is symmetric with a positive Z^I shock
- Using base-year prices eliminates dependence as by Bai, Rios-Rull, and Storesletten (2024)
- Fisher index also eliminates dependence on base year, but it is equivalent in the case of a first-order approximation.
- See Duernecker, Herrendorf, Valentinyi et al. (2017) for a detailed discussion

Back to mapping

Calibration

Calibration: normalizations and standard targets

Targets	Target value	Parameter	Calibrated value/posterior mean
Third group: normalizations			
SS output	1	z_{mc}	0.30
Relative price of services	1	z_{sc}	0.45
Relative price of investment	1	z_i	0.25
Fraction time spent working	0.30	θ_n	0.45
Capacity utilization of nondurables	0.81	A_{mc}	2.6
Capacity utilization of services	0.81	A_{sc}	1.5
Capacity utilization of investment sector	0.81	A_i	3.6
Capital utilization rate	1	σ_b	0.031
Fourth group: standard targets			
Investment share of output	0.20	δ	1.37%
Physical capital to annual output ratio	2.75	α_k	0.31
Labor share of income	0.67	α_n	0.053

Table 8: Calibration: normalization and standard targets. Use posterior mean of the estimated parameters $\sigma, \zeta, \phi, \eta, \nu^R$ and ha.

Details: depreciation

- Over sample, the average annual growth rate of output is 1.8%
- Set $\overline{g} = 0.45\%$ (1.8% annual growth)
- Capital accumulation (ignoring adjustment costs)

$$g\widehat{K}' = (1-\delta)\widehat{K} + g\widehat{I}$$

so that in steady state

$$\delta = 1 - \overline{g} + \frac{I}{K}$$

- Let investment share $\kappa = p_i I/Y = 0.2$ and $p_i K/Y = 2.75(4) = 11$
- Hence, $\delta = 0.2/11 - 0.0045 = 1.37\%$

Details: labor share α_n

- Rearrange FOC for labor demand

$$p_j = (1-\phi)\frac{W_j n_j}{\alpha_n A_j (D_j)^{\phi} F_j}$$

Hence,

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1+\nu^R)$$

where $\nu^R = \nu_j/(F_j)$ and thus labor share is

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi}\frac{C + p_i I}{Y}(1+\nu^R) = \frac{\alpha_n}{1-\phi}(1+\nu^R)$$

so that $\alpha_n = (1-\phi)$labor share$/(1+\nu^R)$

Details: capital share α_k and deprecation parameter σ_b

- $R_j = R$ in steady state
- Note $\beta(\overline{g})^{-\sigma} = 1/(1+r) \Rightarrow \overline{g} - 1 \approx (r - \rho)/\gamma$
- Implies $\rho \approx r - \gamma\overline{g}$ (so we must have $r \geq \gamma\overline{g}$)
- Steady-state Euler

$$
\begin{aligned}
Q &= \beta\overline{g}^{-\gamma}[(1-\delta)Q + R] \Rightarrow \\
(1+r)Q &= (1-\delta)Q + R \\
(r+\delta)Q &= R
\end{aligned}
$$

- Steady-state optimal utilization

$$\sigma_b = \frac{R}{Q} = r + \delta$$

- Combine with steady state Tobin's Q: $p_i/(1-\phi) = Q$ and we find

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

Details: capital share α_k and deprecation parameter σ_b

- Firm optimization yields

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{K_j}(1+\nu^R)$$

- Note

$$\frac{Y_j}{K_j} = \frac{Y}{K} \quad \forall K$$

and hence

$$r + \delta = \alpha_k \frac{Y}{K}(1+\nu^R)$$

so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{K}{Y}$$

Using $r, \delta, K/Y, \nu^R$, we recover α_k

Details: weight of services ω_{sc}

- We pin down the weight of services ω_{sc} as the empirical measure $S_c = Y_{sc}/C$ and set $S_c = 0.65$.
- The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  and plug in S_c, using $\omega_{sc} = S_c$:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

  so that $p_{mc} = p_{sc}$

- Given normalization $p_{sc} = 1$, all consumption goods prices equal unity.

Details: matching technology coefficient A_j

- Given $\Psi_j = A_j D_j^{\phi}$, the matching technology coefficient satisfies

$$A_j = \frac{\Psi_j}{D_j^{\phi}}$$

- Need to find D_j for each j

Details: matching technology coefficient A_j

- We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j \rightarrow$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

- Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Details: matching technology coefficient A_j

- Consider ratio in shopping optimality conditions between m_c and i:

$$\frac{D_{mc}}{D_i} = \frac{p_{mc}}{p_i}\frac{Y_{mc}}{Y_i} = (1-\omega_{sc})\frac{1-I/Y}{I/Y}$$

- Hence,

$$D_{mc} = (1-S_c)(1-I/Y)D$$
$$D_{sc} = S_c(1-I/Y)D$$
$$D_i = (I/Y)D$$

Estimation

Balanced growth and transformation of variables

- Output, consumption, investment, wages, and capital grow at common rate g_t
- Transform each trending variable y_t determined at time t

$$\widehat{y}_t = \frac{y_t}{X_t}$$

so that $\log \widehat{y}_t$ represents log deviation from stochastic trend

- Capital stock K_t is determined at $t-1$, so we deflate by X_{t-1}

$$\widehat{K}_t = \frac{K_t}{X_{t-1}}$$

- Transform preferences to make shopping stationary

$$\Gamma_t = c_t - haC_{t,-1} - X_t \theta_{dt} \frac{d^{1+1/\eta}}{1+1/\eta} - \theta_{nt} X_t \frac{(n_t^a)^{1+1/\nu}}{1+1/\nu}$$

Equations modified by growth

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$C_t^{obs} = \log C_t - \log C_{t-1} + g_t - \overline{g}$$
$$I_t^{obs} = \log I_t - \log I_{t-1} + g_t - \overline{g}$$

- Stationary series

$$N_{jt}^{obs} = \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\}$$
$$p_{i,t}^{obs} = \log p_{i,t} - \log p_{i,t-1}$$
$$util_{j,t}^{obs} = \log util_{j,t} - \log util_{j,t-1}$$

Vector of observable variables

Vector of observables

$$
= \begin{bmatrix} \Delta \log(C_t) \\ \Delta \log(I_t) \\ \Delta \log(N_{ct}) \\ \Delta \log(N_{it}) \\ \Delta \log(util_{ND,t}) \\ \Delta \log(util_{D,t}) \\ \Delta \log(p_{it}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$C_t^{obs} = \log C_t - \log C_{t-1} + g_t - \overline{g}$$
$$I_t^{obs} = \log I_t - \log I_{t-1} + g_t - \overline{g}$$

- Stationary series

$$N_{jt}^{obs} = \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\}$$
$$p_{i,t}^{obs} = \log p_{i,t} - \log p_{i,t-1}$$
$$util_{j,t}^{obs} = \log util_{j,t} - \log util_{j,t-1}, \quad j \in \{mc, i\}$$

Back to stochastic processes

Estimation procedure

- Estimate mode of posterior distribution by maximizing log posterior function (combines priors and likelihood)
- Use Metropolis-Hastings algorithm to sample posterior distribution and to evaluate marginal likelihood of the model
- Mode is used to initialize Markov chain
- Sample over 1 million draws (discard first 30%)
- Hessian defines transition probability that generates new proposed draw
- Check convergence and identification (trace plots)

Back to estimation

On the use of growth rates for estimation

- Many macroeconomic series are difference-stationary
- Growth rates are a model-consistent way to render observables stationary
- Kalman filter uses same transformation on model and data
- Other filters (such as HP filter/Hamilton filter) extract specific frequencies of time series
- Latter may be reasonable for *description* depending on the notion of business cycle

FEVD: breakdown of search demand shocks

Table 9: Forecast error variance decomposition

	e_d	e_{di}
Y	97.23	2.77
SR	94.26	5.74
I	88.83	11.17
p_i	46.65	53.35
n_c	99.67	0.33
n_i	96.38	3.62
$util$	96.92	3.08
D	99.97	0.03
h	98.77	1.23

Table 9: Contribution of components to forecast error variance decomposition of search shocks.

FEVD: breakdown of technology shocks

Table 10: Forecast error variance decomposition

	e_g	e_Z	e_{zI}
Y	31.68	63.30	5.02
SR	48.24	43.87	7.90
I	3.25	74.14	22.62
p_i	0.14	43.91	55.95
n_c	22.23	75.51	2.26
n_i	6.20	61.70	32.10
$util$	0.64	83.26	16.10
D	10.20	76.28	13.52
h	1.34	89.29	9.36

Table 10: Contribution of components to forecast error variance decomposition of technology shocks.

Back to FEVD

BRS as special case

- Model nests BRS by shutting down additional frictions:
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_K = 0$
  - $\theta = 0$
- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^{\phi}$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise: role of capacity utilization data in BRS special case

- Fix $\beta = 0.99, \sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- Total shock processes $\{\theta_d, \theta_n, g, z, z_I\}$
- In contrast to BRS, estimate ϕ and η
- Otherwise use same prior distributions

Table 11: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Role of capacity utilization on parameter estimates

Table 12: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
σ_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 12: Estimation of baseline BRS model with two sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Comparison of volatility and variance decomposition

Table 13: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Std. dev.		
D	1.54	1.69
$util$	0.15	1.49
FEVD of demand shocks θ_d		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 13: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The BRS dataset includes growth rates of output, investment, labor productivity, and the relative price of investment. The second column adds variable total capacity utilization. The second sub-table shows the fraction of the variance decomposition attributable to the demand shock θ_d. See Table 12.

Back to Quantitative analysis

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